Appendix to Non-Parametric Unfolding of Binary Choice Data

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This appendix is a supplement to “Non-Parametric Unfolding of Binary Choice Data” (*Political Analysis*, 8:211-237, 2000). Section A1 shows how the new normal vector is obtained from the singular value decomposition of the p by s matrix $\boldsymbol{\Psi}^*$ constructed from the legislator coordinates. Sections A2 and A3 report Monte-Carlo studies of the cutting plane procedure and legislative procedure, respectively, with voting error. Finally, section A4 shows a Monte-Carlo study of the unfolding algorithm with missing data and voting error in two and three dimensions.

Appendix

A1. The sth singular vector, $\underline{\boldsymbol{\theta}}_s$, is the Normal Vector, $\underline{\mathbf{n}}_j^{\#}$

Recall that the singular value decomposition of the p by s matrix $\boldsymbol{\Psi}^*$ is

$$\boldsymbol{\Psi}^* = \mathbf{U\Lambda\Theta'}$$

and that the current estimate of the cutting hyperplane is obtained by applying the Eckart-Young theorem to $\boldsymbol{\Psi}^*$ (Eckart and Young, 1936). Namely, the best fitting hyperplane of rank s-1 through a set of points of rank s is found by performing a singular value decomposition of the matrix of points, $\mathbf{U\Lambda\Theta'}$, inserting a zero in place of the sth singular value on the diagonal of $\boldsymbol{\Lambda}$, and remultiplying. That is:

$$\mathbf{V} = \mathbf{U\Lambda^{\#}\Theta'}$$

where $\Lambda^{\#}$ is an s by s diagonal matrix identical to Λ except for the replacement of the sth singular value by zero. By construction, the p by s matrix $\mathbf{V}$ has rank s-1.

Let $\underline{\mathbf{n}}_j^{\#}$ be the normal vector of the hyperplane defined by $\mathbf{V}$ such that $\underline{\mathbf{n}}_j^{\#\prime}\, \underline{\mathbf{n}}_j^{\#} = 1$ and let $\underline{\boldsymbol{\theta}}_s$ be the sth singular vector of Θ. It is easy to show that $\underline{\mathbf{n}}_j^{\#} = \underline{\boldsymbol{\theta}}_s$ (or its reflection, $\underline{\mathbf{n}}_j^{\#} = -\underline{\boldsymbol{\theta}}_s$). To see this, note that by the definition of an orthogonal matrix:

$$\underline{\boldsymbol{\theta}}_s'\Theta = (\mathbf{0, 0, 0, \ldots, 1})$$

That is, the inner product of $\underline{\boldsymbol{\theta}}_s$ with the other s-1 singular vectors in $\boldsymbol{\Theta}$ is zero. Hence:

$$\Lambda^{\#}\boldsymbol{\Theta}'\underline{\boldsymbol{\theta}}_s = \underline{\mathbf{0}}_s$$

and

$$\mathbf{V}\underline{\boldsymbol{\theta}}_s = \mathbf{U}\Lambda^{\#}\boldsymbol{\Theta}'\underline{\boldsymbol{\theta}}_s = \underline{\mathbf{0}}_p \qquad \text{(A1)}$$

Where $\underline{\mathbf{0}}_s$ and $\underline{\mathbf{0}}_p$ are vectors of zeroes of length s and p respectively. By construction, equation (A1) is simply a restatement of the definition of a plane. The normal vector to the plane, $\mathbf{V}$, is $\underline{\boldsymbol{\theta}}_s$ (or its reflection, $-\underline{\boldsymbol{\theta}}_s$). Hence, $\underline{\mathbf{n}}_j^{\#} = \underline{\boldsymbol{\theta}}_s$.

A2. Monte-Carlo Studies of the Cutting Plane Procedure with Voting Error

Recovery of the Normal Vector

Similar to the experiments shown in Table 1, 100 legislators and 500 pairs of policy points were randomly drawn from a uniform distribution through the unit hypersphere. The policy points were randomly drawn but in such a way so as to produce an average majority margin of about 67 percent. Error was introduced by making the legislator choices probabilistic such that the further a legislator is from the cutting plane, the less likely the legislator will make a voting error. Specifically, an indirect utility function (McFadden, 1976) was created for each legislator -- $u_{ij\vartheta} + \varepsilon_{ij\vartheta}$ – where $u_{ij\vartheta}$ is the deterministic portion of the utility function for choice ϑ=Yea, Nay, and $\varepsilon_{ij\vartheta}$ is the stochastic portion. The deterministic portion is assumed to be an exponential function of the negative of the squared distance from the legislator to the "y" and "n" alternatives and ε_{ijy} and ε_{ijn} were drawn from the Normal, Uniform, and Logit distributions, respectively.

Table A1 shows that the procedure does a good job correctly classifying the true roll call choices and recovering the true normal vectors – especially at the 15 percent error level which is the approximate level of the error found in the U.S. Congressional roll call data.[1] Finally, as one would expect, increasing the number of legislators increases the accuracy of the recovery.

Table A1 about Here

When error is present the cutting plane procedure converges very quickly. An example is shown in Figure A1 that uses the same configuration of legislator ideal points as Figure 3. The choices of 78 of the 435 legislators have been modified so that they are "errors" – "N's" on the "Y" side of the true cutting line and "Y's" on the "N" side of the true cutting line. The cutting plane procedure converges on the 30th iteration as shown in Panel D. As shown by Panels B and C, in the error case the converged cutting plane may not be the one that maximizes classification – however, it will invariably be very close to the optimal cutting plane. This is easily dealt with by simply storing the iteration record and using the normal vector corresponding to the best classification. This works very well in practice.

Figure A1 about Here

Bootstrapped Standard Errors For the Elements of the Normal Vector

Given the legislator coordinates, **X**, and their votes on the jth roll call, $\underline{\mathbf{t_j}}$ **,** standard errors for the estimated normal vector, $\mathbf{n_j}^*$, can be obtained via a simple bootstrapping analysis. In this context the rows of **X** and the corresponding elements in $\underline{\mathbf{t_j}}$ are sampled with replacement. In a simple binary limited dependent variable context, let $\mathbf{X}^\#$ be the matrix **X** bordered by a column of ones, then

$$\underline{\mathbf{t_j}} = \mathbf{X}^\#\underline{\beta} + \underline{\varepsilon} ,$$

In a Probit or logit analysis, if the estimated β's for the independent variables, $\beta_1 , \beta_2 , \ldots , \beta_s$, are normalized so that their sum of squares is equal to one, then they constitute a normal vector to a plane upon which the choice probabilities are exactly .5/.5 and the intercept term, β_0, is the cutting point, $\mathbf{m_j}$***.** In this context **X** is a *fixed set of numbers*. However, in the roll call context, **X** is estimated from the roll calls. Consequently, the bootstrap standard errors reported below should be regarded as *measuring the stability* of the cutting plane procedure. In an LDV context where **X** is a matrix of independent variables, they are more akin to real standard errors.

The bootstrap analysis is performed in the following manner. First, the rows of **X** and the corresponding elements in $\underline{\mathbf{t}}_{\mathbf{j}}$ are sampled with replacement to form 100 matrices (that is, the sampling is by legislator with replacement). Second, the cutting plane procedure is applied to each of the 100 matrices. Finally, the standard errors are obtained by computing the sum of squared differences between the actual normal vector from the original data, $\mathbf{n}_{\mathbf{j}}^{*}$, and the 100 normal vectors from the bootstrap trials, dividing by 100 and taking the square root.

The matrix **X** was constructed by randomly drawing legislators from a uniform distribution through the unit hypersphere. Experiments were conducted using 50, 100, and 500 legislators, respectively. The elements of the true normal vector were all set equal to $\sqrt{\frac{1}{s}}$ so that the s dimensions would be equally salient and a cutting point was chosen along the normal vector to produce either a 50-50 margin or an 80-20 margin, respectively. Voting error was generated in the same way as the experiments reported in Table A1, however, only normally distributed error was used in the experiments reported in Table A2. The cutting plane procedure was applied to the matrix with error to obtain the estimated normal vector, $\mathbf{n}_{\mathbf{j}}^{*}$, and then the standard errors were computed for this $\mathbf{n}_{\mathbf{j}}^{*}$ using the bootstrap method described in the previous paragraph. This entire process was repeated 50 times producing 50s (50 times s) standard errors. The mean and standard deviation of these 50s standard errors are reported for differing values of p (50, 100, 500), s (2, 3, 10), and margin (50-50, 80-20) in Table A2.

Table A2 about Here

For example, the first row of Table A2 shows the results for 50 legislators in 2 dimensions with 15 percent voting error (due to rounding, this was either 7 or 8 of the 50 legislators making voting errors). The average of the 100 bootstrapped standard errors (50 experiments times 2 dimensions) was 0.104 for the 50-50 margin votes (each true roll call was 25 Yea and 25 Nay) and 0.177 for the 80-20 margin votes (each true roll call was 40 Yea and 10 Nay). The standard deviations of these two means were 0.029 and 0.065 respectively.

The average standard errors in Table A2 show that lopsided roll calls are less precisely estimated than close roll calls, and roll calls with a smaller number of legislators the less precisely estimated than roll calls with a larger number. Neither of these findings is a surprise. The bottom line is that the cutting plane procedure is very stable at realistic levels of observations and levels of error.

Empirical Comparisons With Probit

Table A3 shows the results of applying Probit and the cutting plane procedure to the Spector and Mazzeo (1980) “Grade” data used by Greene (1993, pp.658-659) to analyze Manski’s Maximum Score Estimator (Manski, 1975, 1985; Manski and Thompson, 1986). The probit coefficients and their standard errors are identical to those reported by Greene (1993, p. 646). The standardized probit coefficients and the estimated normal vector from the cutting plane procedure are very similar. The standard errors for the elements of the normal vector estimated by the cutting plane procedure were obtained by a simple bootstrapping analysis. The Spector and Mazzeo dataset was sampled by observation with replacement (that is, the *rows* of the data matrix were sampled with replacement) to form 100 matrices and the cutting plane procedure was applied to each of the 100 matrices. The standard errors were obtained by computing the sum of squared differences between the actual normal vector from the original data and the 100 normal vectors from the bootstrap trials, dividing by 100 and taking the square root.[2] This is the same approach used in the Monte Carlo work reported above.

Table A3 about Here

The pattern of "significance" for the three non-parametric cutting plane coefficients is the same as that for the Probit coefficients. Monte-Carlo work with artificial data suggests that the cutting plane coefficients will have nearly identical patterns of significance (using bootstrapping) with those produced by a Probit analysis when the underlying error distribution is symmetric.

Table A4 shows a second empirical comparison of the cutting plane procedure and Probit analysis. The sample is 231 Republican members of the House of Representatives[3] and the dependent variable is whether or not they signed up as co-sponsors of a minimum wage increase.[4] The independent variables are the two dimensional W-NOMINATE scores (Poole and Rosenthal, 1997) computed from votes taken in 1995 and some characteristics of representatives' congressional districts (percent rural, percent Black, and median family income). (The independent variables were put in standard deviation form to facilitate comparisons.) The standardized Probit coefficients and the cutting plane coefficients are very close – the simple Pearson correlation is .961. Substantively, the coefficients in Table A4 indicate that Republican moderates from poorer, urban districts support raising the minimum wage. Once again, the pattern of "significance" for the non-parametric cutting plane coefficients is the same as that for the Probit coefficients.

Table A4 about Here

A3. . Monte-Carlo Studies of the Legislative Procedure With Voting Error

Recovery of the Legislative Coordinates

Table A5 is organized in the same fashion as Table A1. Not surprisingly, as the number of cutting planes increases with the error level held fixed, the precision of the recovery of the legislators increases dramatically. Even at the very high error level of 25 percent, with 500 roll calls in two or three dimensions the recovery of the legislator coordinates is very good.

Table A5 about Here

The legislative procedure is very stable. This is shown by the small standard deviations for correct classifications and the r-squares. In addition, the gap between the average worst r-square and the average best r-square for the s dimensions is not very large. For example, for 500 roll calls in 3 dimensions, the average worst r-square between the true and reproduced legislator coordinates was .968 and the average best r-square was .985. In other words, *on average*, the three r-squares computed between the corresponding three dimensions ranged between .968 and .985.

Bootstrapped Standard Errors For the Legislator Coordinates

Given the normal vectors, **N,** the cutpoints, the q $\mathbf{m_j}$'s, and legislator i's votes on the q roll calls, $\underline{\mathbf{t}}_{\mathbf{i}}$ **,** standard errors for the estimated legislator coordinates, $\underline{\mathbf{x}}_{\mathbf{i}}^{*}$, can be obtained via a simple bootstrapping analysis. Two types of bootstrapping experiments are reported below. In the first, similar to the analysis of the cutting plane procedure above, the stability of the legislative procedure is assessed by assuming that **N** is fixed. In this context the rows of **N** and the corresponding $\mathbf{m_j}$'s and elements in $\underline{\mathbf{t}}_{\mathbf{i}}$ are sampled with replacement. The resulting standard errors for the entries of **X** are a useful descriptive measure of the stability of the legislative procedure.

In the second set of experiments, a matrix of roll calls is created and the unfolding algorithm is run until convergence to get the estimated legislator coordinates, **X***. Then 100 matrices are formed from the original roll call matrix by drawing roll calls with replacement and the unfolding algorithm is run on all 100 matrices to produce 100 estimated legislator matrices. The standard errors for the legislator coordinates are then computed from these 100 bootstrapped estimates. These standard errors are good descriptive measures of the stability of the unfolding algorithm as a whole and mimic a real world application of the unfolding procedure.

The bootstrap analysis to assess the stability of the legislative procedure is performed in the following manner. First, the rows of **N** and the corresponding $\mathbf{m_j}$'s and elements in $\underline{\mathbf{t}}_{\mathbf{i}}$ are sampled with replacement to form 100 matrices (that is, the sampling is by roll call cutting plane with replacement). Second, the legislative procedure is applied to each of the 100 matrices. Finally, the standard errors are obtained by computing the sum of squared differences between the actual legislator coordinates from the original data, $\underline{\mathbf{x}}_{\mathbf{i}}^{*}$, and the 100 $\underline{\mathbf{x}}_{\mathbf{i}}$'s from the bootstrap trials, dividing by 100 and taking the square root.

The bootstrap analysis to assess the stability of the unfolding algorithm as a whole is performed in the following manner. An artificial roll call matrix is created with a given level of voting error and average majority margin that closely approximates actual congressional roll call data. The unfolding algorithm is then applied to this matrix to get the target legislator configuration, **X***. From the roll call matrix, 100 roll call matrices are formed by sampling roll calls with replacement. The unfolding algorithm is applied to each of the 100 roll call matrices and the standard deviation of the 100 estimates for each legislator for each dimension was computed. Each of the 100 estimated configurations was rotated using Schonemann's (1966) method to best fit the target configuration, **X***, to remove any arbitrary rotation. (Empirically, the rotation of the configurations was extremely small.) The standard errors are obtained by computing the sum of squared differences between the actual legislator coordinates from the original data, $\underline{\mathbf{x}}_{\mathbf{i}}^{*}$, and the 100 $\underline{\mathbf{x}}_{\mathbf{i}}$'s from the bootstrap trials (after rotation), dividing by 100 and taking the square root.

Figure A2 shows the results of these bootstrapping experiments (type 1 and type 2, respectively) for 100 legislators and 500 roll calls in 2, 3, and 10 dimensions. The classification error introduced was 18% and the average majority margin was 68-32 for all the experiments. Figure A2A shows the results of both types of bootstrapping for 2 dimensions, figure A2B shows the results for 3 dimensions, and figure A2C 10 dimensions.

Figure A2 about Here

For example, in the 2 dimensional experiments, about 12 percent of the 300 (100 legislators times 3 dimensions) type 1 standard errors and about 3 percent of the type 2 standard errors were between 0.03 and 0.04. The distributions of the standard errors for 2 and 3 dimensions are piled up to the left of the corresponding ones for 10 dimensions. This makes sense because the number of legislators and roll calls are held fixed while the number of dimensions is increased. Recall that q roll calls in s dimensions create a maximum of $\sum_{k=0}^{s}\binom{q}{k}$ regions in the space. This number explodes as the number of dimensions increases so that it is quite likely that in high dimensional spaces there are multiple regions close to each other with the same correct classification for a legislator. This geometry is almost certainly responsible for the slightly larger standard errors from the bootstrapping experiments. Nevertheless, even in 10 dimensions the bulk of the standard errors are reasonably small.

Figure A3 shows type 2 bootstrapped standard errors for the 98 Senators shown in Figure 6. Because there were only 255 roll calls in the 85th Senate with minority percentages of 2.5 percent or better, the standard errors are somewhat larger than those shown in Figure A2A. In addition, the two dimensions were not equally salient as they are constrained to be in the Monte Carlo work. Although the second dimension played a significant role during this period – especially on civil rights related issues – the first dimension accounts for the bulk of the roll call votes (Poole and Rosenthal, 1997). Consequently, the standard errors are much lower on the first dimension. Eighty of 98 Senators have standard errors of less than .10. The standard errors for the second dimension are larger reflecting the fact that the bulk of the cutting lines are between 60 and 120 degrees (see Figure 6). Even so, 72 of 98 Senators have standard errors of less than .15, which is small relative to the 2-unit diameter of the space. The standard errors tend to be larger for those Senators near the rim of the space.

Figure A3 about Here

A4. . Monte-Carlo Study of the NonParametric Unfolding Algorithm With Missing Data and Voting Error

Table A6 shows a set of experiments with and without error at various levels of missing data. Configurations of 100 legislators and 500 roll calls in 2 and 3 dimensions were randomly generated in the same fashion as those used in the Monte-Carlo experiments shown in Table 4. Error was introduced into the choices by making them probabilistic (see Appendices A2 and A3 above). An error level of about 20 percent was chosen because it is somewhat above the approximate level of error in U.S. congressional roll call data. Matrix entries were randomly removed and the remaining entries were then analyzed by the algorithm in one through five dimensions. The upper part of Table A6 shows two-dimensional experiments at four different levels of missing data with and without error, and the lower part shows three-dimensional experiments. Each randomly produced matrix was analyzed at each level of missing data so that the same 10 matrices for two or three dimensions (with varying levels of missing entries) are being averaged in each row of the upper or lower parts of the Table.

Table A6 about Here

The accuracy of the recovery of the legislator configuration is quite good and only begins to fall off at 70 percent missing entries. With perfect data the procedure unambiguously finds the true dimensionality. With error there are clear “elbows” at the true dimensionality. The tendency for the correct classification to increase with the percentage of missing data is due to the fact that with more missing data there are fewer roll call cutting planes and hence a legislator’s position is not as constrained as it is with complete data. Indeed, the average largest distance to a cutting plane increases with the level of missing data. This tends to increase the correct classification and decrease the correlation between the true and reproduced legislator configurations. In any event, the results shown in Table A6 suggest that the algorithm will perform well with real world data at realistic levels of missing entries. In particular, with 20 percent missing data there is no appreciable deterioration in performance.

Appendix References

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Figure A1A. Error Example

78 Errors With True Cutting Line

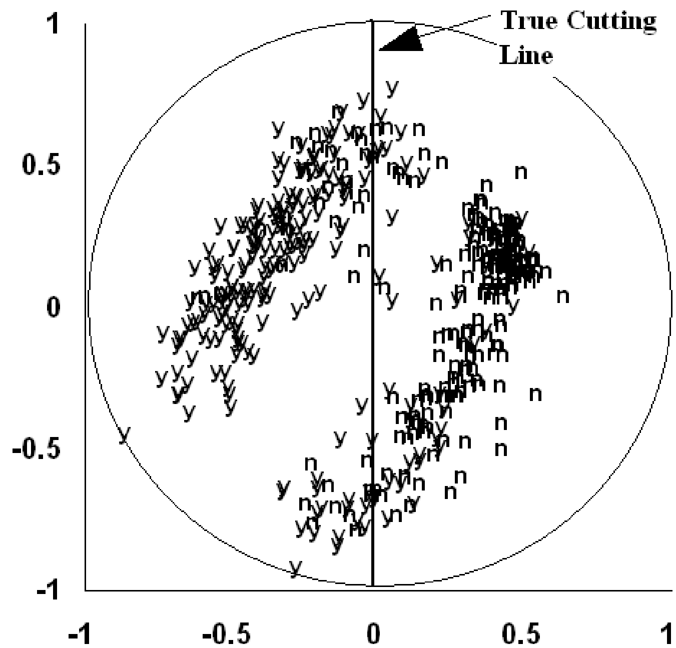

Figure A1B. Error Example

73 Errors at 10th Estimate (Minimum)

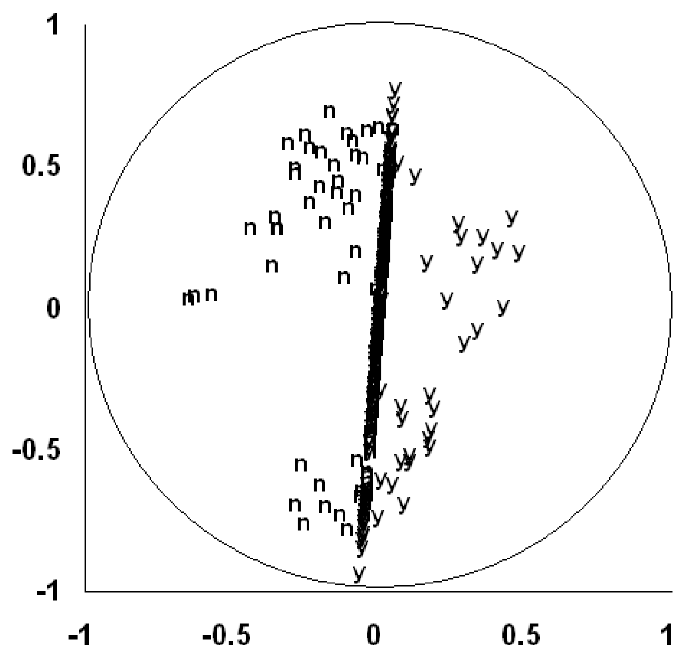

Figure A1C. Error Example
73 Errors at 20th Estimate (Minimum)

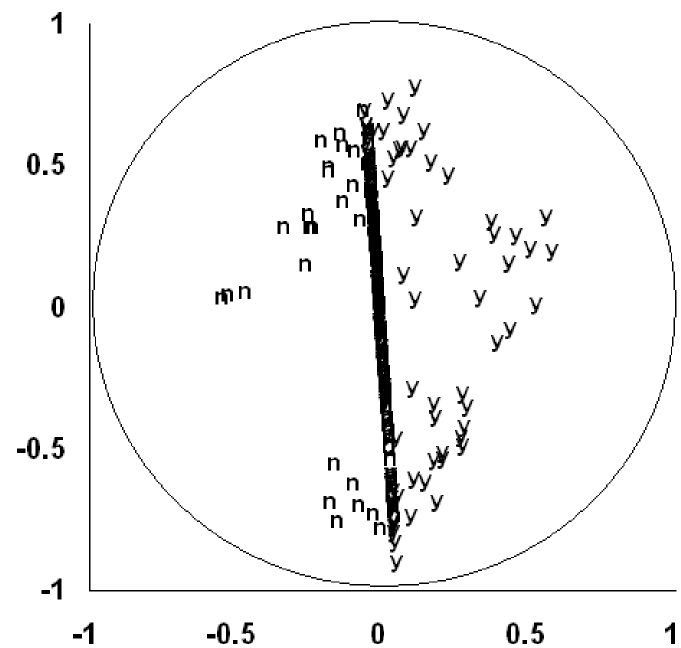

Figure A1D. Error Example
75 Errors at 30th Estimate (Converged)

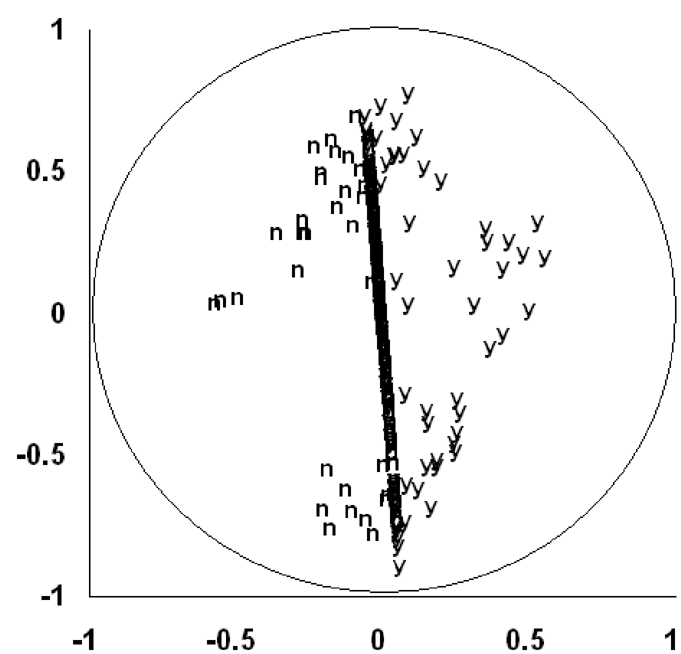

Figure A2A: Bootstrap Standard Errors For Legislative Procedure

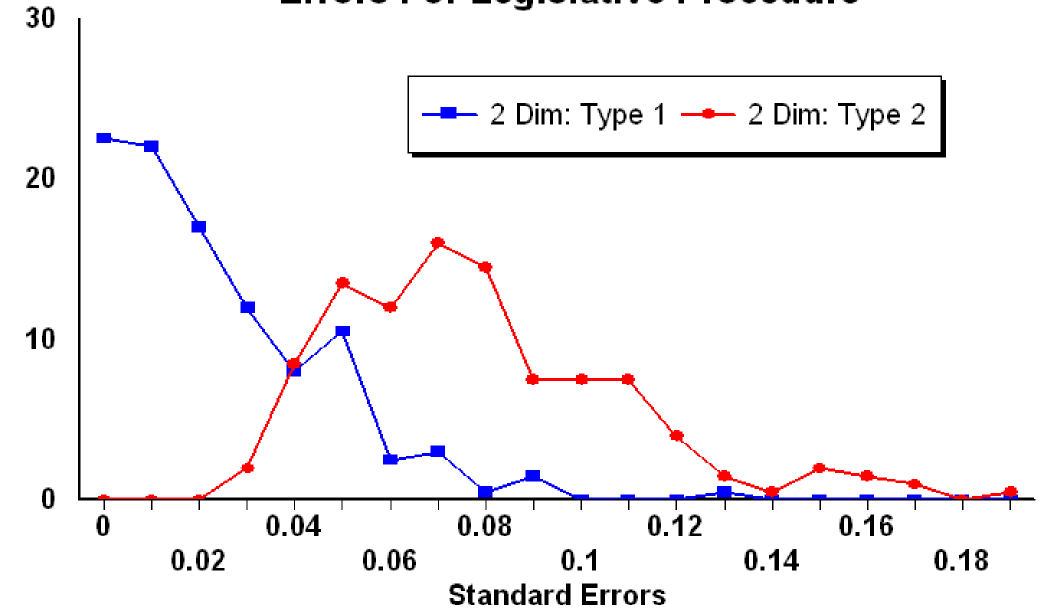

Figure A2B: Bootstrap Standard Errors For Legislative Procedure

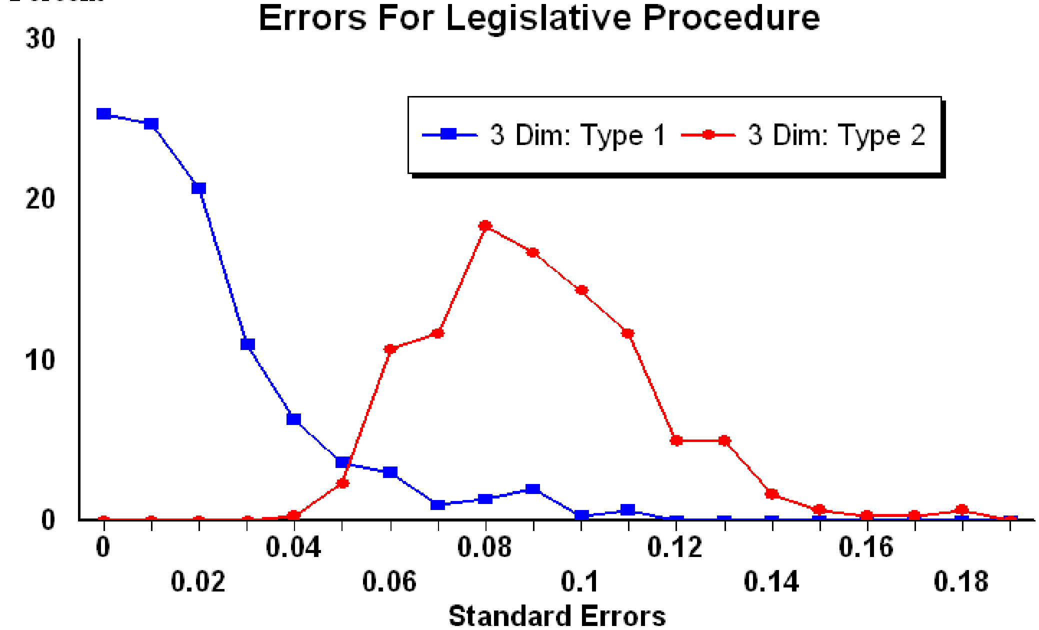

Figure A2C: Bootstrap Standard Errors For Legislative Procedure

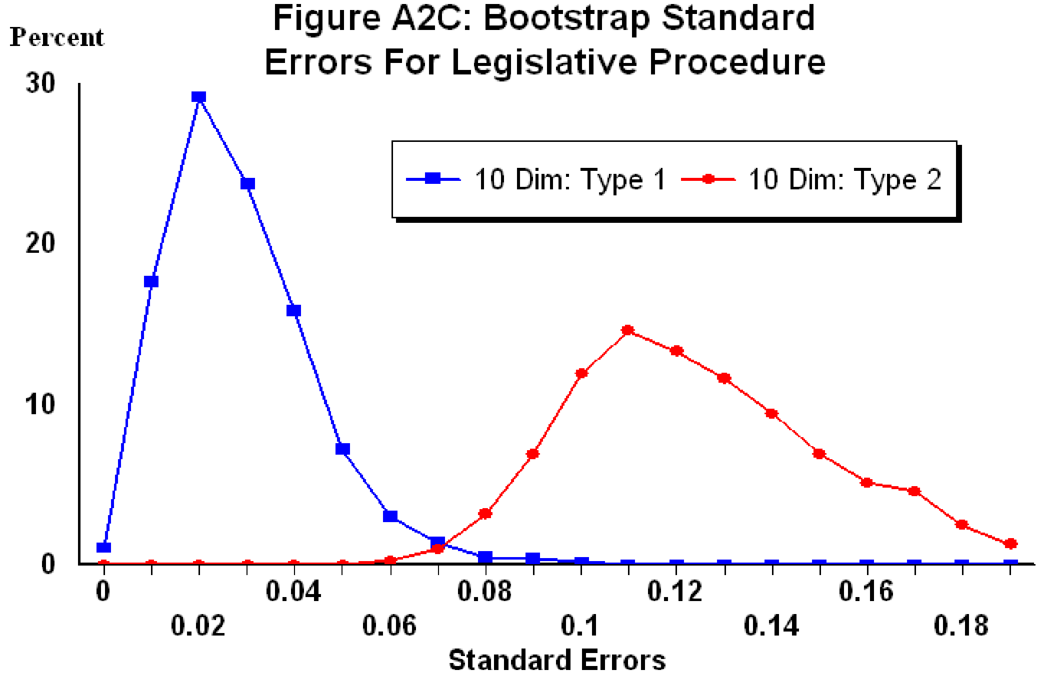

Figure A3: Bootstrap Standard Errors For 85th Senate

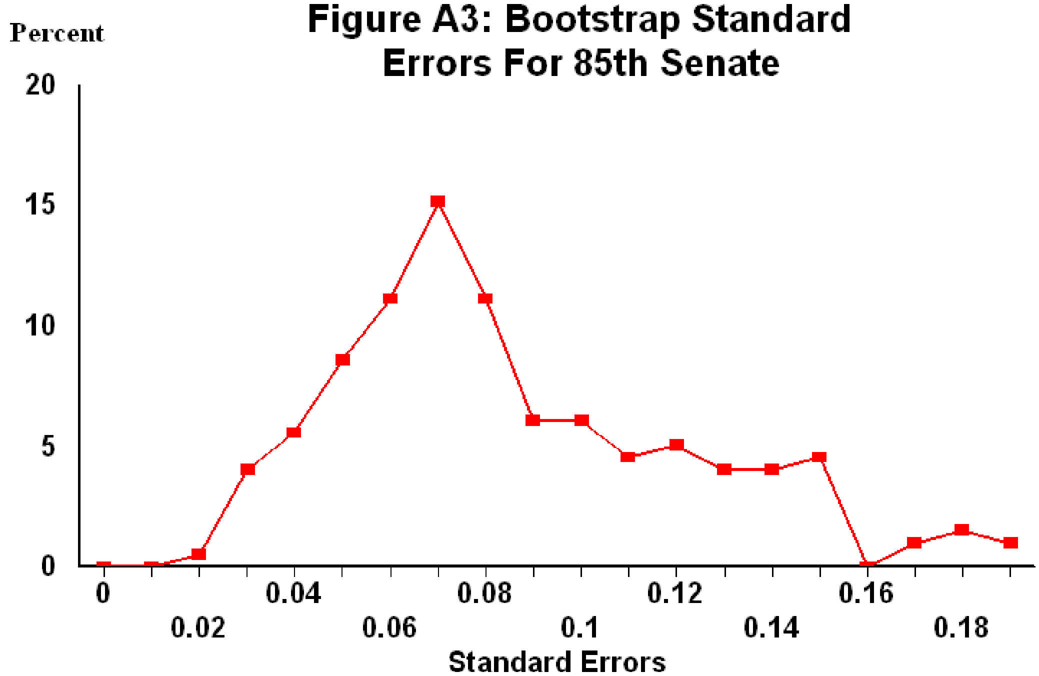

Table A1
Monte-Carlo Tests of Cutting Plane Procedure
500 Votes With Normal, Uniform, and Logit Error
(Each Entry Average of 10 Trials, Standard Deviations in Parentheses)

S	P	Average Percent Error	Average Majority Margin	Average *Percent* Correctly Classified Obs.[a]	Average Percent Correctly Classified True[b]	Average Fit With True Normal Vectors All[c]	Average Fit With True Normal Vectors 10% Min.[d]
1	100	24.9 (0.4)	65.9 (0.6)	77.9 (0.2)	91.6 (0.5)	.840 (.035)	.894 (.015)
1	100	15.7 (2.6)	66.5 (0.8)	86.5 (0.6)	94.6 (0.3)	.906 (.026)	.939 (.007)
2	100	25.5 (0.2)	64.1 (0.2)	78.8 (0.3)	90.3 (0.3)	.951 (.004)	.952 (.003)
2	100	15.0 (0.6)	66.4 (0.8)	89.7 (0.5)	94.2 (0.2)	.979 (.004)	.986 (.001)
3	100	25.1 (0.3)	64.8 (0.6)	80.5 (0.4)	88.9 (0.3)	.913 (.008)	.918 (.006)
3	100	14.3 (0.3)	68.0 (0.5)	89.8 (0.2)	93.0 (0.2)	.954 (.004)	.969 (.002)
3	25	14.8 (0.5)	67.6 (0.6)	93.4 (0.6)	88.8 (0.4)	.890 (.011)	.909 (.013)
3	50	14.8 (0.4)	66.8 (0.7)	90.8 (0.4)	91.0 (0.3)	.934 (.008)	.952 (.003)
3	200	14.5 (0.2)	67.2 (0.8)	88.3 (0.2)	94.4 (0.2)	.970 (.002)	.980 (.002)
3	100	15.0[e] (0.4)	66.9 (0.7)	89.3 (0.4)	92.4 (0.2)	.960 (.004)	.968 (.002)
3	100	15.4[f] (0.5)	68.2 (0.5)	88.9 (0.5)	92.4 (0.3)	.952 (.003)	.965 (.003)

[a] Average correct classification of observed roll call data.

[b] Average correct classification of true roll call data.

[c] For one dimension, average Spearman correlation between estimated rank order and true rank order of midpoints. For more than one dimension, average cosine between estimated and true normal vectors.

[d] For one dimension, average Spearman correlation between estimated rank order and true rank order of midpoints for roll call with at least 10 percent or better in the minority. For more than one dimension, average cosine computed between estimated and true normal vectors for roll calls with at least 10 percent or better in the minority.

[e] Uniform distribution error.

[f] Logit distribution error.

Table A2

Monte-Carlo Tests of Cutting Plane Procedure
Boot-Strapped Standard Errors For Elements of the Normal Vector

Each Entry the Average of 50 experiments
Each Experiment 100 Bootstrap Trials

			True Margin	
S	**P**	**Percent Error**	**50 - 50 Average Standard Error**	**80 - 20 Average Standard Error**
2	50	15	0.104 (0.029)	0.177 (0.065)
2	100	15	0.072 (0.019)	0.106 (0.040)
2	500	15	0.036 (0.007)	0.047 (0.008)
2	50	25	0.154 (0.050)	0.221 (0.102)
2	100	25	0.106 (0.029)	0.187 (0.100)
2	500	25	0.059 (0.009)	0.075 (0.019)
3	50	15	0.137 (0.046)	0.179 (0.066)
3	100	15	0.091 (0.023)	0.126 (0.036)
3	500	15	0.039 (0.006)	0.052 (0.008)
3	50	25	0.194 (0.062	0.258 (0.092)

3	100	25	0.143 (0.036)	0.199 (0.078)
3	500	25	0.062 (0.009)	0.083 (0.014)
10	50	15	0.172 (0.039)	0.201 (0.054)
10	100	15	0.104 (0.023)	0.122 (0.026)
10	500	15	0.040 (0.005)	0.055 (0.008)
10	50	25	0.195 (0.042)	0.227 (0.057)
10	100	25	0.134 (0.027)	0.170 (0.035)
10	500	25	0.062 (0.008)	0.085 (0.011)

Table A3

Empirical Comparison of Probit and Cutting Plane Procedure
Spector and Mazzeo "Grade" Data
(Source: Green, 1993, p. 639-640)

Dependent Variable = 1 if Examination Grade Improved with New Teaching Method for Economics

Variable	Probit Coefficient	Std. Error	Standardized Probit Coefficients[d]	Cutting Plane Coefficients	Boot-Strapped Std. Error[e]
Constant	-7.452	2.542	---	---	---
GPA[a]	1.626	.694	.752	.787	.207
TUCE[b]	0.052	.084	.024	.001	.032
PSI[c]	1.426	.595	.659	.617	.316

Probit Log Likelihood = -12.819
Percent Correctly Classified By Probit = 81.3 (26 of 32)
Percent Correctly Classified By Cutting Plane Procedure = 87.5 (28 of 32)

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[a] **Grade Point Average**
[b] **Score on a pretest that indicates entering knowledge of the material**
[c] **Indicator variable: = 1 if student exposed to new teaching method**
[d] **The sum of the squared coefficients equals 1**
[e] **Based upon 100 trials**

Table A4

Empirical Comparison of Probit and Cutting Plane Procedure
22 Republican Defectors on Minimum Wage: April 1996

Dependent Variable = 1 if Support Raising Minimum Wage; 0 if Oppose
Independent Variables Expressed in Standard Deviation Form
Margin 22 - 209

Variable	Probit Coefficient	Std. Error	Standardized Probit Coefficients[d]	Cutting Plane Coefficients	Boot-Strapped Std. Error[e]
Constant	1.722	0.186	---	---	---
W-NOM[a] 1st Dimension	12.211	3.481	.911	.876	0.238
W-NOM[a] 2nd Dimension	0.785	2.951	.059	.266	0.347
Rural[b]	4.157	2.163	.310	.338	0.160
Black[b]	0.337	1.926	.025	.051	0.151
Median Income[c]	3.568	2.484	.266	.212	0.220

Probit Log Likelihood = -54.101
Percent Correctly Classified By Probit = 90.9 (210 of 231)
Percent Correctly Classified By Cutting Plane Procedure = 91.8 (212 of 231)
Correlation Between Cutting Plane Coefficients and Standardized Probit Coefficients = .961

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[a] **Unadjusted W-NOMINATE scores range from -1.0 to +1.0**
[b] **Unadjusted data expressed as a percentage**
[c] **Unadjusted data expressed in dollars**
[d] **The sum of the squared coefficients equals 1**
[e] **Based upon 100 trials**

Table A5

Monte-Carlo Tests of Legislator Procedure
100 Legislators With Normal, Uniform, and Logit Error
(Each Entry Average of 10 Trials, Standard Deviations in Parentheses)

S	Q	Average Percent Error	Average Majority Margin	Average *Percent* Correctly Classified Obs.[a]	Average Percent Correctly Classified True[b]	Average Worst Leg. R-Square[c]	Average Best Leg. R-Square[d]
1	500	25.4 (0.6)	63.7 (0.6)	77.8 (0.6)	91.3 (0.4)	---	.985 (.002)
1	500	15.9 (0.3)	66.9 (0.6)	86.4 (0.4)	94.4 (0.2)	---	.985 (.012)
2	500	24.4 (0.7)	64.9 (0.3)	79.2 (0.6)	91.3 (0.2)	.967 (.016)	.984 (.006)
2	500	15.2 (0.3)	68.1 (0.7)	87.6 (0.4)	94.2 (0.2)	.971 (.013)	.991 (.003)
3	500	25.5 (0.2)	65.6 (0.5)	78.3 (0.3)	90.1 (0.3)	.943 (.016)	.972 (.008)
3	500	16.2 (0.3)	67.1 (0.6)	86.6 (0.4)	93.4 (0.2)	.968 (.012)	.985 (.003)
3	50	16.2 (0.2)	68.5 (2.1)	90.4 (0.7)	89.6 (0.9)	.725 (.052)	.819 (.018)
3	100	16.1 (0.4)	67.8 (1.1)	88.6 (0.3)	91.2 (0.4)	.835 (.016)	.888 (.018)
3	250	16.1 (0.4)	66.9 (0.8)	87.1 (0.3)	92.7 (0.3)	.930 (.016)	.955 (.008)
3	500	14.8[e] (0.5)	67.1 (0.7)	88.0 (0.4)	93.5 (0.2)	.968 (.009)	.982 (.007)
3	500	15.3[f] (0.2)	68.1 (0.9)	87.4 (0.3)	93.4 (0.2)	.968 (.011)	.986 (.004)

[a] Average correct classification of observed roll call data.

[b] Average correct classification of true roll call data.

[c] R-Squares computed between true and reproduced legislator coordinates. The number shown is the average of the worst r-squares across the 10 trials.

[d] R-Squares computed between true and reproduced legislator coordinates. The number shown is the average of the best r-squares across the 10 trials.

[e] Uniform distribution error.

[f] Logit distribution error.

Table A6
Monte-Carlo Tests: Non-Parametric Unfolding of Binary Choice Matrices With Missing Data
(Each Entry Average of 10 Trials, Standard Deviations in Parentheses)
2 Dimensions, 100 Legislators, 500 Votes

Percent Missing	Average Percent Error	Average Majority Margin	Percent Correct 1 Dim.	Percent Correct 2 Dim.	Percent Correct 3 Dim.	Percent Correct 4 Dim	Percent Correct 5 Dim.	R^2 1st	R^2 2nd
0	0	65.5	91.4	99.9	100.0	100.0	100.0	.982	.948
		(0.7)	(0.9)	(0.0)	(0.0)	(0.0)	(0.0)	(.005)	(.005)
20	0	65.5	91.6	99.9	100.0	100.0	100.0	.983	.949
		(0.7)	(1.5)	(0.0)	(0.0)	(0.0)	(0.0)	(.005)	(.009)
50	0	65.9	91.6	99.9	100.0	100.0	100.0	.975	.938
		(0.8)	(0.9)	(0.0)	(0.0)	(0.0)	(0.0)	(.005)	(.020)
70	0	66.5	92.1	99.9	100.0	100.0	100.0	.964	.920
		(0.8)	(0.7)	(0.0)	(0.0)	(0.0)	(0.0)	(.007)	(.013)
0	20.3	64.6	79.3	84.0	85.1	86.0	86.6	.968	.953
	(0.6)	(0.8)	(0.7)	(0.4)	(0.3)	(0.3)	(0.3)	(.007)	(.004)
20	20.3	64.8	79.3	84.7	86.0	86.8	87.5	.963	.936
	(0.6)	(0.8)	(0.8)	(0.4)	(0.3)	(0.3)	(0.3)	(.005)	(.012)
50	20.3	65.3	80.5	85.9	87.5	88.7	89.5	.949	.925
	(0.6)	(0.8)	(0.5)	(0.4)	(0.3)	(0.4)	(0.4)	(.012)	(.012)
70	20.3	66.1	81.1	87.6	89.7	91.3	92.8	.929	.915
	(0.6)	(0.9)	(0.8)	(0.5)	(0.5)	(0.5)	(0.5)	(.009)	(.012)

3 Dimensions, 100 Legislators, 500 Votes

Percent Missing	Average Percent Error	Average Majority Margin	Percent Correct 1 Dim.	Percent Correct 2 Dim.	Percent Correct 3 Dim.	Percent Correct 4 Dim	Percent Correct 5 Dim.	R^2 1st	R^2 2nd	R^2 3rd
0	0	66.7	85.3	92.7	100.0	100.0	100.0	.985	.976	.957
		(0.7)	(0.4)	(0.8)	(0.0)	(0.0)	(0.0)	(.005)	(.009)	(.010)
20	0	66.6	85.5	92.3	100.0	100.0	100.0	.981	.976	.958
		(0.8)	(0.5)	(0.9)	(0.0)	(0.0)	(0.0)	(.002)	(.004)	(.013)
50	0	67.0	85.6	92.9	99.9	100.0	100.0	.962	.946	.924
		(0.7)	(0.5)	(0.9)	(0.0)	(0.0)	(0.0)	(.009)	(.013)	(.015)
70	0	68.0	86.3	93.4	99.9	100.0	100.0	.932	.911	.883
		(0.4)	(0.3)	(0.8)	(0.0)	(0.0)	(0.0)	(.012)	(.019)	(.022)
0	22.8	61.3	73.8	77.7	81.2	81.9	82.6	.982	.980	.979

	(0.7)	(0.4)	(0.6)	(0.5)	(0.3)	(0.3)	(0.3)	(.007)	(.009)	(.007)
20	22.8	61.7	74.1	78.3	81.9	82.6	83.3	.981	.977	.974
	(0.6)	(0.5)	(0.7)	(0.5)	(0.3)	(0.3)	(0.2)	(.007)	(.007)	(.011)
50	22.8	62.4	74.9	79.9	83.4	84.3	85.3	.969	.966	.952
	(0.5)	(0.5)	(0.7)	(0.4)	(0.3)	(0.2)	(0.3)	(.013)	(.017)	(.017)
70	22.8	63.4	76.4	82.1	85.6	87.1	88.5	.947	.938	.908
	(0.6)	(0.4)	(0.7)	(0.6)	(0.4)	(0.3)	(0.4)	(.021)	(.022)	(.020)

Appendix Endnotes

[1] The first two dimensions estimated by NOMINATE classify about 85 percent of the roll call choices during the post World War II period (Poole and Rosenthal ,1997, ch. 2).

[2] Note that in all the cutting plane analyses, the means of the variables in the grade data were subtracted so that the sum of the columns of the matrix was zero. Then the whole matrix was multiplied by a constant to scale the matrix so that it was within a hypersphere of radius one.

[3] In April, 1996, there were 236 Republicans in the House. The 5 party switchers – Laughlin (TX), Parker (MS), Hayes (LA), Deal (GA), and Tauzin (LA) – were excluded from the analysis producing an n of 231. Campbell (R-CA), who won a special election to replace Mineta (D-CA), is included.

[4] The source for the co-sponsors is "Who You Calling 'Moderate?'", by Bob Balkin, *PoliticsUSA*, at www.politicsusa.com, Wednesday, April 24, 1996.